### ****Q1: Combinatorics****

#### ****a. Sample Space of Die + Alphabet****

* **Die outcomes**: 6 (1, 2, ..., 6).
* **Alphabet letters**: 26 (A to Z).
* **Total sample points**: 6×26=1566×26=156.

**Answer**: 156156​.

#### ****b. Basketball Team Selection****

* **Positions**: 5 (distinct).
* **Players**: 8 (each can play any position).
* **Ways to fill**: Permutation P(8,5)=8×7×6×5×4=6720*P*(8,5)=8×7×6×5×4=6720.

**Answer**: 67206720​.

#### ****c. Permutations of "INFINITY"****

* **Letters**: I, N, F, I, N, I, T, Y.
* **Total letters**: 8.
* **Repeated**: I (3 times), N (2 times).
* **Distinct permutations**:

8!3!×2!=403206×2=3360.3!×2!8!​=6×240320​=3360.

**Answer**: 33603360​.

### ****Q2: Conditional Probability****

#### ****Given****:

* **Juniors**: 10 (3 A’s).
* **Seniors**: 30 (10 A’s).
* **Graduate students**: 10 (5 A’s).
* **Total students**: 10+30+10=5010+30+10=50.
* **Total A’s**: 3+10+5=183+10+5=18.

#### ****a. P(Senior | A)****

P(Senior∣A)=A’s by seniorsTotal A’s=1018=59.*P*(Senior∣*A*)=Total A’sA’s by seniors​=1810​=95​.

**Answer**: 5995​​.

#### ****b. P(A | Junior)****

P(A∣Junior)=A’s by juniorsTotal juniors=310.*P*(*A*∣Junior)=Total juniorsA’s by juniors​=103​.

**Answer**: 310103​​.

### ****Q3: Set Theory & Probability****

#### ****Given****:

* **Total students**: 100.
* **Math (M)**: 54.
* **History (H)**: 69.
* **Both (M ∩ H)**: 35.

#### ****a. P(M ∪ H)****

P(M∪H)=P(M)+P(H)−P(M∩H)=54100+69100−35100=88100=2225.*P*(*M*∪*H*)=*P*(*M*)+*P*(*H*)−*P*(*M*∩*H*)=10054​+10069​−10035​=10088​=2522​.

**Answer**: 22252522​​.

#### ****b. P(Neither M nor H)****

P(Neither)=1−P(M∪H)=1−88100=12100=325.*P*(Neither)=1−*P*(*M*∪*H*)=1−10088​=10012​=253​.

**Answer**: 325253​​.

#### ****c. P(H but not M)****

P(H∖M)=P(H)−P(M∩H)=69100−35100=34100=1750.*P*(*H*∖*M*)=*P*(*H*)−*P*(*M*∩*H*)=10069​−10035​=10034​=5017​.

**Answer**: 17505017​​.

### ****Q4: Bayes’ Theorem****

#### ****Given****:

* **Engineer 1 (E₁)**: 70% jobs, error rate 0.02.
* **Engineer 2 (E₂)**: 30% jobs, error rate 0.04.
* **Observed**: Serious error occurred.

#### ****Find****: P(E2∣Error)P(E2​∣Error).

1. **Total error probability**:

P(Error)=P(E1)×P(Error∣E1)+P(E2)×P(Error∣E2)=0.7×0.02+0.3×0.04=0.026.*P*(Error)=*P*(*E*1​)×*P*(Error∣*E*1​)+*P*(*E*2​)×*P*(Error∣*E*2​)=0.7×0.02+0.3×0.04=0.026.

1. **Apply Bayes’ Theorem**:

P(E2∣Error)=P(E2)×P(Error∣E2)P(Error)=0.3×0.040.026=0.0120.026=613≈0.4615.*P*(*E*2​∣Error)=*P*(Error)*P*(*E*2​)×*P*(Error∣*E*2​)​=0.0260.3×0.04​=0.0260.012​=136​≈0.4615.

Since P(E2∣Error)>P(E1∣Error)*P*(*E*2​∣Error)>*P*(*E*1​∣Error), **Engineer 2 is more likely**.

**Answer**: Engineer 2Engineer 2​.

### ****Summary of Answers****

| **Question** | **Answer** |
| --- | --- |
| Q1a | 156 |
| Q1b | 6720 |
| Q1c | 3360 |
| Q2a | 5995​ |
| Q2b | 310103​ |
| Q3a | 22252522​ |
| Q3b | 325253​ |
| Q3c | 17505017​ |
| Q4 | Engineer 2 |

**Key Formulas Used**:

* Permutations with repetitions: n!n1!⋅n2!⋯nk!*n*1​!⋅*n*2​!⋯*nk*​!*n*!​.
* Conditional probability: P(A∣B)=P(A∩B)P(B)*P*(*A*∣*B*)=*P*(*B*)*P*(*A*∩*B*)​.
* Bayes’ Theorem: P(E2∣Error)=P(E2)⋅P(Error∣E2)P(Error)*P*(*E*2​∣Error)=*P*(Error)*P*(*E*2​)⋅*P*(Error∣*E*2​)​.